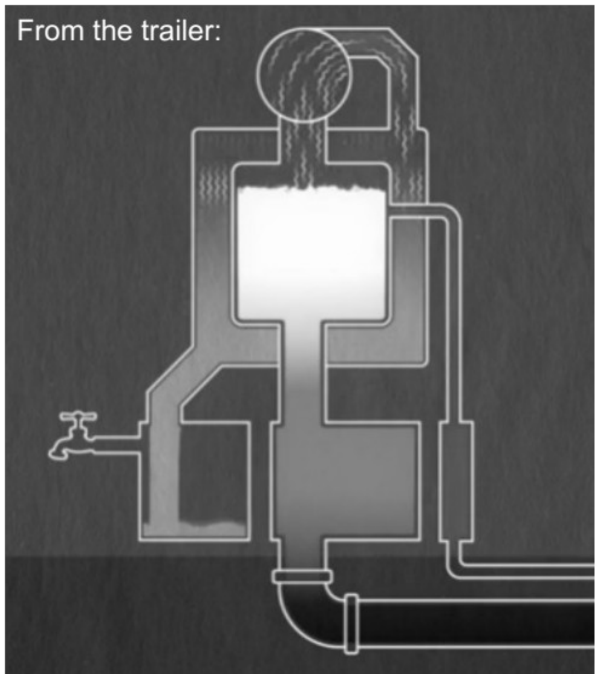
ChE-304 Problem Set 5

Week 5

Problem 1

After watching the video introducing the Slingshot water purification technology, you might have realized that this is just a purification system based on boiling water, compressing it and condensing the (now pure) steam.

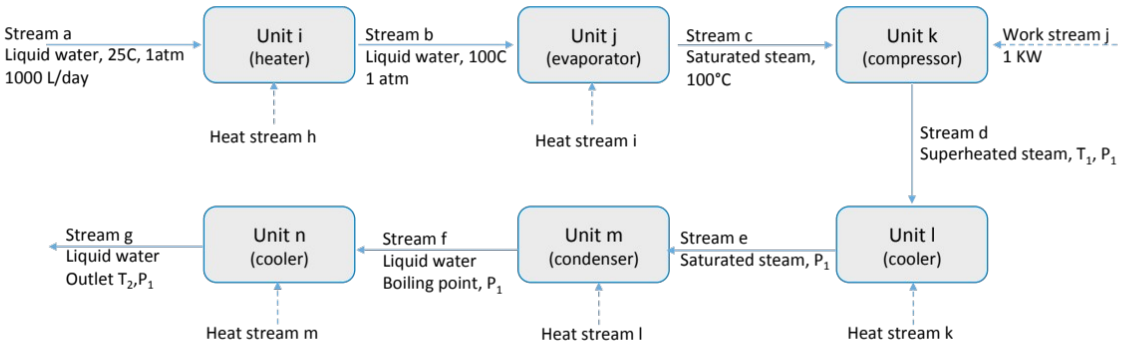


List all material and work streams and draw the system, following the example of Figure 2.4A. Also list **all** the specifications. As we will see later in the course, it is often convenient to split liquid heating, gas heating and phase change into 3 separate units.

We know (from watching the whole movie) that the system uses 1KW electricity to treat 1000 L/day.

Note: the cooling and condensing of the compressed steam is used to heat up the boiling water but we will worry about that later…. For now, list the streams independently.

Solution:



We assume that a pure water stream enters the system at room temperature (25°C) and atmospheric pressure. This allows us to treat the system as a one-component system (we can likely ignore the impurities from a thermodynamic point of view). The water then gets heated to 100°C boils and becomes saturated steam at 1 atm. The steam then gets compressed. We assume that this compression happens isentropically and adiabatically (see problem 3). The steam then cools to its saturation temperature (at some higher pressure P_1 determined in problem 4), and then condenses. The liquid then further cools to an acceptable temperature (T_2) for consumption.

Problem 2

Calculate the properties (T, P and $\dot{V}_{steam}$) of the slingshot steam after compression using 1 kW of electricity for 1000 L per day of liquid water. Assume that the compression is adiabatic and that 100% of the electrical work ends up as mechanical compression work. You can also assume that steam acts like an ideal gas.

Cp (steam) = 1.97 kJ/ (kg K)

Cv (steam) = 1.5 kJ/ (kg K)

Solution:

For an adiabatic compression, we have:

$$\Delta U = W = C_V \left(T_2 - T_1 \right)$$

$$\dot{M}_{steam} = 1000 \frac{L}{day} = 0.0116\, kg/sec$$

$$\dot{W} = 1000\, J/sec$$

The input work for 1 kg of steam is:

$$W = 86.4\, kJ\ per\, kg\, steam$$

$$T_2 = T_1 + \frac{W}{C_V} = 100°C + \frac{86.4\, kJ}{1.50\, \frac{kJ}{K}} = 157.6$$

$°C$

$$k = \frac{C_P}{C_V} = 1.31$$

$$\left(\frac{T_2}{T_1} \right) = \left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}} \rightarrow P_2 = P_1 \left(\frac{T_2}{T_1} \right)^{\frac{k}{k-1}} = 1\, atm \left(\frac{420.5}{373} \right)^{\frac{1.31}{0.31}} = 1.83\, atm$$

$$\dot{V}_{steam,1} = \dot{n} \frac{RT}{P} = \frac{11.6}{18} \frac{8.314 * 373}{101325} = \frac{0.0196\, m^3}{sec} = 19.6\, L/sec$$

$$\dot{V}_{steam,2} = \dot{V}_{steam,1} \left(\frac{P_1}{P_2} \right)^{1/k} = 11.4\, L/sec$$

Problem 3

Can you calculate the entropy of 1 mol of a liquid mixture of X and a salt at 120°C and 0.7 MPa where the molar fraction of salt is: $x_{salt}=0.05$? You can assume an ideal solution.

<u>Parameters:</u>

$R=8.314\,J\,K^{-1}\,mol^{-1}$
$S_{salt}=100\,J/(K\,mol)$ Molar entropy for the salt at 120°C and 0.7 MPa
S_X^0 = 188.84 $J\,K^{-1}\,mol^{-1}$ (Standard entropy of gaseous X at 25°C and 0.1 MPa)

Cp equation:

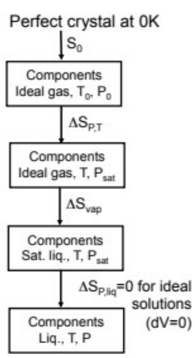
$$C_P(T)=A_X+B_X T+C_X T^2+D_X T^3+\frac{E_X}{T^2}\text{ , where } T=temperature\in K$$

Cp equation coefficients, for X in the vapor phase:
A = 30 $J\,K^{-1}\,mol^{-1}$ B = 0 $J\,K^{-2}\,mol^{-1}$ C = 0 $J\,K^{-3}\,mol^{-1}$
D = 0 $J\,K^{-4}\,mol^{-1}$ E = 82'000 $J\,K\,mol^{-1}$

$\Delta H_{vap}^{100°C}$= 40.6 kJ/mol (Enthalpy of vaporization, X at 100°C)
$T_C=$¿ 647 K $P_C=$¿22 MPa (Critical temperature, pressure of X)
$P_{sat,X\,120°C}=0.2\,MPa$ (Saturation pressure for X at 120°C)

Solution

We use:



$$dS=\frac{C_p}{T}dT-\frac{R}{P}dP$$

$$\Delta S_{T,P_{sat}}=\left[A_\alpha \ln\left(\frac{T_1}{T_0}\right)+B_\alpha(T_1-T_0)+\frac{C_\alpha(T_1^2-T_0^2)}{2}+\frac{D_\alpha(T_1^3-T_0^3)}{3}-\frac{E_\alpha}{2}\left(\frac{1}{T_1^2}-\frac{1}{T_0^2}\right)\right]-\left[R*\ln\left(\frac{P_{sat}}{P_0}\right)\right]$$

$$¿30\ln\left(\frac{393}{298}\right)+0+0-0-\frac{82000}{2}\left(\frac{1}{393^2}-\frac{1}{298^2}\right)-8.314\ln\left(\frac{0.2}{0.1}\right)=3.1\,J\,mol^{-1}K^{-1}$$

$$\Delta H_{vap,120°C}=\Delta H_{vap,100°C}\left(\frac{T_C-120°C}{T_C-100°C}\right)^{0.38}=40'600\left(\frac{254}{274}\right)^{0.38}=39'447\,J/mol$$

$$\Delta S_{vap,120°C}=\frac{\Delta H_{vap,120°C}}{T}=\frac{39'447}{393}=100.4\,J\,K^{-1}mol^{-1}$$

7 points

$$S_{L,X}(T,P)=S_\alpha^0+\int_{T_0}^{T}\frac{Cp_\alpha(T')}{T'}dT'-Rln\frac{P_{sat}}{P_0}-\Delta S_{vap,\alpha}(T)=188.8+3.1-100.4=91.5\,J\,K^{-1}mol^{-1}$$

4 points

For an ideal system, there is still an entropy of mixing:

$$S❑_{mixing}=-R\left(x_{salt}\ln x_{salt}+x_X\ln x_X\right)=-1*8.314*(0.05\ln 0.05+0.95\ln 0.95)=1.65\,Jmol^{-1}K^{-1}$$

$$S_{ideal,mix}=x_X S_{L,X}+x_{salt}S_{salt}+S❑_{mixing}=0.95*91.5+0.05*100+1.65=93.6\,J\,K^{-1}mol^{-1}$$